Lecture 6

7. HARMONIC CURRENT CIRCUIT CALCULATION METHODS

7.1. Peculiarities of Harmonic Current Circuit Calculation

As mentioned above, the method of direct solution of differential equations can be used to calculate the simplest electric circuits, for example, circuits of the first order. However, even in the case of such circuits we can see that application of the complex amplitude method significantly simplifies the calculations. In calculation of complicated branched circuits of the second and higher orders such as a resonant transistor amplifier, in which the transistor in the linear mode is replaced by the equivalent substitutional circuit, or a three-phase asynchronous motor power circuit, in which each phase of the motor is a second-order circuit, the method of direct solution of differential equations is practically impossible to apply. The only method that can be used to calculate circuits of this kind is the complex amplitude method [11–13].

Along with harmonic current circuits, direct current circuits are widely used in practice. The calculation of such circuits requires solution of algebraic equation systems only. As compared to direct current circuits, the calculation of harmonic current circuits has the following features:

1) dependence of the currents and voltages on the frequency;

2) dependence of impedances and conductances on the frequency;

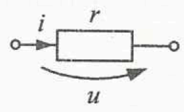
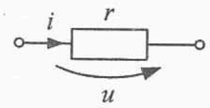
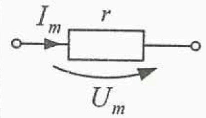
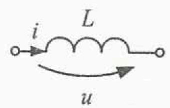
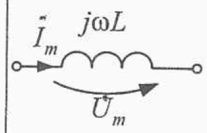
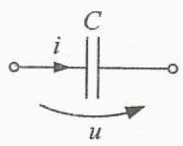
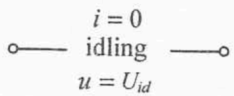
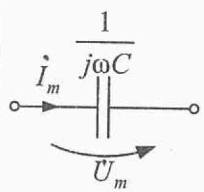
3) the need to take into account the phase relationships between the currents and voltages;

4) circulation of reactive power between the active and passive elements of the circuit;

5) resonant phenomena.

The dependence of the resistances on the frequency can be represented by Table 7.1. As we can see from Table 7.1 the inductance L in a harmonic current circuit is equivalent to the resistance $j\omega L$, and in a direct current circuit it is equivalent to a short-circuit as the frequency in a direct current circuit is $\omega = 0$. The capacity C in a harmonic current circuit is equivalent to the resistance $1/j\omega C$, and in a direct current circuit it is equivalent to a broken circuit (no load).

Table 7.1

Element	Circuit	*DC* circuit	*AC* circuit
Resistance	 i, r, u	 i, r, u	 $\dot{I}_m$, r, $\dot{U}_m$
Inductance	 i, L, u	 i short circuit $u = 0$	 $\dot{I}_m$, $j\omega L$, $\dot{U}_m$
Capacitance	 i, C, u	 $i = 0$ idling $u = U_{id}$	 $\dot{I}_m$, $\frac{1}{j\omega C}$, $\dot{U}_m$

The active resistance r has the same resistance r in both direct and harmonic current circuits.

7.2. The Equivalent Complex Circuit

In (3.4) the general procedure for calculating a harmonic current circuit by the complex amplitude method was presented.

In this case, as it was pointed out, we have to build an *equivalent complex circuit* (ECC). Let us consider the procedure of obtaining the ECC.

In Fig. 7.1 a three-phase *asynchronous motor* (AM) power circuit is shown.

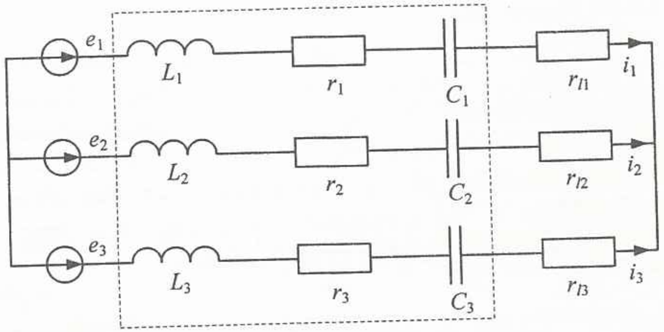


Fig. 7.1

Here the three motor windings are represented by the resistances r_1, r_2, r_3, inductances L_1, L_2, L_3 and capacitances C_1, C_2, C_3.

The power-supply circuit is represented by the three-phase voltage source e_1, e_2, e_3. The load circuit is represented by the resistances r_{l1}, r_{l2}, r_{l3}.

The voltage sources e_1, e_2, e_3 form a three-phase system. In terms of complex amplitudes their images can be written as

$$\begin{cases} e_1 = E_m\cos\omega t \doteqdot \dot{E}_{m1} = E_m; \\ e_2 = E_m\cos\left(\omega t - \dfrac{2\pi}{3}\right) \doteqdot \dot{E}_{m2} = E_m e^{-j\frac{2\pi}{3}}; \\ e_3 = E_m\cos\left(\omega t + \dfrac{2\pi}{3}\right) \doteqdot \dot{E}_{m3} = E_m e^{j\frac{2\pi}{3}}. \end{cases} \quad (7.1)$$

The complex impedances of the phases of the circuit are:

$$\begin{cases} Z_1 = r_1 + r_{l1} + j\left(\omega L_1 - \dfrac{1}{\omega C_1}\right); \\ Z_2 = r_2 + r_{l2} + j\left(\omega L_2 - \dfrac{1}{\omega C_2}\right); \\ Z_3 = r_3 + r_{l3} + j\left(\omega L_3 - \dfrac{1}{\omega C_3}\right). \end{cases} \quad (7.2)$$

From (7.1) and (7.2) we get the ECC (Fig. 7.2).

Here the currents are:

$$\begin{cases} \dot{I}_{m1} \doteqdot i_1 = I_{m1}\cos(\omega t + \psi_1); \\ \dot{I}_{m2} \doteqdot i_2 = I_{m2}\cos(\omega t + \psi_2); \\ \dot{I}_{m3} \doteqdot i_3 = I_{m3}\cos(\omega t + \psi_3). \end{cases}$$

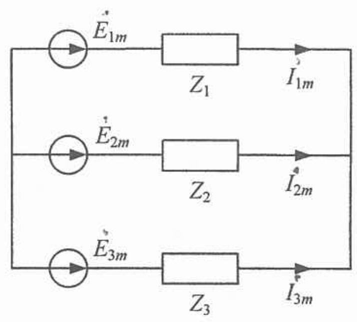


Fig. 7.2

Hence, the procedure of obtaining the ECC is the following:

1) select the conventionally positive directions of the currents in the branches of the original circuit;

2) use the same trigonometric representation for the instantaneous values of harmonic currents, voltages and EMF: all of them should be expressed either in terms of sine or in terms of cosine functions;

3) replace the instantaneous values of voltages and currents, voltages and EMF by their images in terms of complex amplitudes; as it takes place, the directions of the voltages and currents in the ECC coincide with the directions of these values in the original circuit.

Thus, the order of circuit calculation by the complex amplitude method can be defined as follows:

1) building an equivalent complex circuit (ECC);

2) setting up a system of algebraic equations in complex form and solve it;

3) moving from the images of unknown quantities in complex form to the originals in real-valued form (obtaining the instantaneous values of these quantities);

4) verifying the correctness of calculation using the active-power balance and reactive-power balance conditions.

Consider the main methods of setting up and solving complex equations of the circuit.

7.3. The Kirchhoff Equation Method

The method is based on direct application of Kirchhoff's current and voltage laws. Here, the maximum number of Kirchhoff's current law equations is set up. The remaining equations are set up on the basis of Kirchhoff's voltage law. Currents in the branches are independent values. The number of equations in the system equals the number of branches in the network. We can define the following calculation procedure by the Kirchhoff equation method:

1) select the conventionally positive directions of currents in the branches;

2) select the independent nodes of the network; select the node in which the greatest number of branches converge as the basic node;

3) set up q–1 Kirchhoff's current law equations for the independent nodes (q — total number of the network nodes);

4) select the independent loops of the network; remember that a branch with an ideal current source does not form a separate loop;

5) select the path-tracing directions of the loops;

6) set up $p - q + 1$ Kirchhoff's voltage law equations for the independent loops (p — number of the network branches);

7) solve the obtained system of equations of Kirchhoff's current and voltage laws together; find the currents in the branches; if the signs of the calculated currents are negative in some of the branches, the actual direction of the currents in these branches is opposite to the selected one;

8) using the found values of currents in the branches and their known resistances, determine the voltages in these branches.

7.4. The Loop Current Method

The Kirchhoff equation method requires setting up and the solution of a system of equations the number of which is equal to the number of the branches in the network. To simplify the process, we can reduce the number of the system of equations breaking the calculation into two stages. At the first stage, intermediate, auxiliary variables called loop currents are calculated. At the second stage, currents in the branches are calculated via the loop currents. The loop currents are referred to as determining variables because currents in the branches are determined through them. Hence, the loop current method is considered to be a method of determining variables. The essence of the loop current method consists in setting up and solving a system of Kirchhoff's voltage law equations. Such equations are set up for independent loops.

Consider the network of an electric circuit shown in Fig. 7.3.

Convert the current source $\dot{J}_{m2}$ into the voltage source $\dot{E}_{m2} = \dot{J}_{m2} Z_2$ (Fig. 7.4).

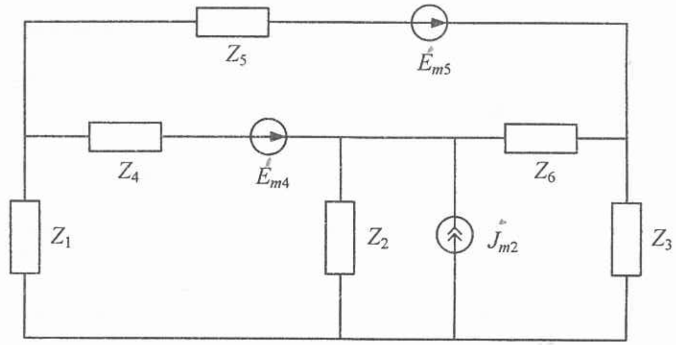


Fig. 7.3

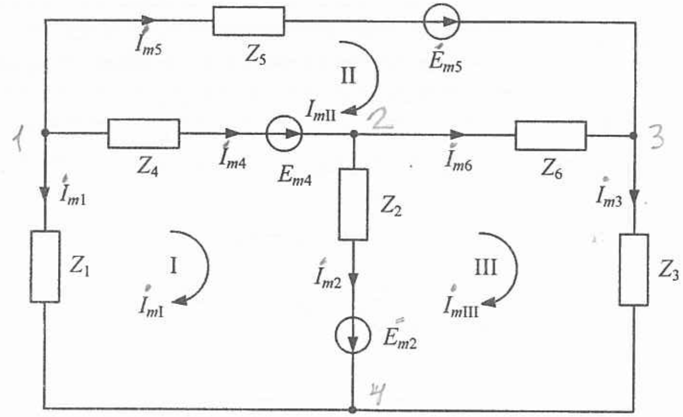


Fig. 7.4

Select the conventionally positive directions of currents in the branches $\dot{I}_{m1} - \dot{I}_{m6}$. We will denote the nodes of the network by 1–4. We get the graph of the network (Fig. 7.5). Select the tree of the graph by the branches $\dot{I}_{m2}, \dot{I}_{m4}, \dot{I}_{m6}$ (the ribs of the tree are solid lines). Point out the chords of the graph (the dotted lines). Denote the chords by $\dot{I}_{mI} - \dot{I}_{mIII}$. Each of the chords together with the edges of the graph forms an independent loop. These loops are shown in Fig. 7.4 and denoted by I–III. The currents of the chords $\dot{I}_{mI} - \dot{I}_{mIII}$ are called the loop currents of the loops I–III in Fig. 7.4. The directions of the loop currents are shown by arrows.

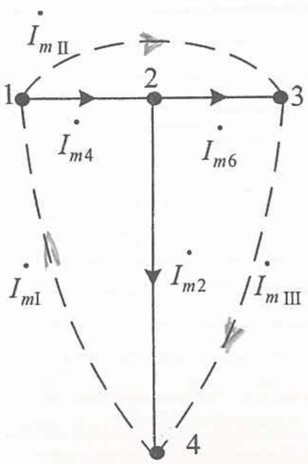


Fig. 7.5

We can see from the graph in Fig. 7.5 that the currents $\dot I_{m1} - \dot I_{m6}$ in the branches of the network in Fig. 7.4 can be expressed in terms of the contour currents $\dot I_{m\mathrm{I}} - \dot I_{m\,\mathrm{III}}$

$$\begin{cases} \dot I_{m1} = -\dot I_{m\mathrm{I}};\ \dot I_{m2} = \dot I_{m\mathrm{I}} - \dot I_{m\mathrm{III}};\ \dot I_{m3} = \dot I_{m\mathrm{III}}; \\ \dot I_{m4} = \dot I_{m\mathrm{I}} - \dot I_{m\mathrm{II}};\ \dot I_{m5} = \dot I_{m\mathrm{II}};\ \dot I_{m6} = \dot I_{m\mathrm{III}} - \dot I_{m\mathrm{II}}. \end{cases} \tag{7.3}$$

But the number of loop currents, as well as the number of independent circuits, is less than the number of branch currents. Hence, the order of the system of equations is reduced and the calculation is simplified.

Determine the loop currents in the network in Fig. 7.4. Kirchhoff's voltage law equations for loops I – III can be written as:

$$\begin{cases} \dot I_{m\mathrm{I}} Z_1 + \left(\dot I_{m\mathrm{I}} - \dot I_{m\mathrm{II}}\right) Z_4 - \dot E_{m4} + \left(\dot I_{m\mathrm{I}} - \dot I_{m\mathrm{III}}\right) Z_2 - \dot E_{m2} = 0; \\ \dot I_{m\mathrm{II}} Z_5 - \dot E_{m5} + \left(\dot I_{m\mathrm{II}} - \dot I_{m\mathrm{III}}\right) Z_6 + \dot E_{m4} + \left(\dot I_{m\mathrm{II}} - \dot I_{m\mathrm{I}}\right) Z_4 = 0; \\ \dot E_{m2} + \left(\dot I_{m\mathrm{III}} - \dot I_{m\mathrm{I}}\right) Z_2 + \left(\dot I_{m\mathrm{III}} - \dot I_{m\mathrm{II}}\right) Z_6 + \dot I_{m\mathrm{III}} Z_3 = 0. \end{cases}$$

or

$$\begin{cases} (Z_1 + Z_2 + Z_4)\dot I_{m4} - Z_4 \dot I_{m\mathrm{II}} - Z_2 \dot I_{m\mathrm{III}} = \dot E_{m2} + \dot E_{m4}; \\ -Z_4 \dot I_{m\mathrm{I}} + (Z_4 + Z_5 + Z_6)\dot I_{m\mathrm{II}} - Z_6 \dot I_{m\mathrm{III}} = \dot E_{m5} - \dot E_{m4}; \\ -Z_2 \dot I_{m\mathrm{I}} - Z_6 \dot I_{m\mathrm{II}} + (Z_2 + Z_3 + Z_6)\dot I_{m\mathrm{III}} = -\dot E_{m2}. \end{cases} \tag{7.4}$$

Here the values

$$Z_{11} = Z_1 + Z_2 + Z_4, Z_{22} = Z_4 + Z_5 + Z_6, Z_{33} = Z_2 + Z_3 + Z_6$$

are called the intrinsic impedances of the I-st, II-nd and III-rd loops respectively.

Thus, the intrinsic impedance of a loop is the sum of the impedances of the branches included in this loop. The values

$$Z_{12} = Z_{21} = -Z_4, Z_{13} = Z_{31} = -Z_2, Z_{23} = Z_{32} = -Z_6$$

are called mutual impedances between the I-st and II-nd, I-st and III-rd, II-nd and III-rd loops respectively. Thus, the mutual impedance between two loops represents the sum of the impedances of the branches included simultaneously in both circuits, taken with the inverse sign. The EMFs

$$\dot E_{m\mathrm{I}} = \dot E_{m2} + \dot E_{m4},\ \dot E_{m\mathrm{II}} = \dot E_{m5} - \dot E_{m4},\ \dot E_{m\mathrm{III}} = -\dot E_{m2}$$

are called loop EMFs of the I-st, II-nd and III-rd loops respectively. They represent the algebraic sum of EMFs included in the branches of the loop and taken with a plus sign if the direction of the EMF coincides with the selected direction of the loop path-tracing, and with a minus sign if the direction of the EMF is opposite to the direction of the loop path-tracing.

In the general case of N independent loops we can write the system of equations

$$\begin{cases} Z_{11} \dot I_{m\mathrm{I}} + Z_{12} \dot I_{m\mathrm{II}} + \cdots + Z_{1N} \dot I_{mN} = \dot E_{m\mathrm{I}}; \\ Z_{21} \dot I_{m\mathrm{I}} + Z_{12} \dot I_{m\mathrm{II}} + \cdots + Z_{2N} \dot I_{mN} = \dot E_{m\mathrm{II}}; \\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots \\ Z_{N1} \dot I_{m\mathrm{I}} + Z_{N2} \dot I_{m\mathrm{II}} + \cdots + Z_{NN} \dot I_{mN} = \dot E_{mN}. \end{cases}$$

Or in matrix form

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \cdot \begin{bmatrix} \dot I_{m\mathrm{I}} \\ \dot I_{m\mathrm{II}} \\ \vdots \\ \dot I_{mN} \end{bmatrix} = \begin{bmatrix} \dot E_{m\mathrm{I}} \\ \dot E_{m\mathrm{I}} \\ \vdots \\ \dot E_{mN} \end{bmatrix}. \tag{7.5}$$

That is

$$\dot Z \dot I = \dot E,$$

where

$$Z=\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{21} \\ \cdots & \cdots & \cdots & \cdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \text{ — loop-impedance matrix (LIM);}$$

$$\dot{I}=\begin{bmatrix} \dot{I}_{m\mathrm{I}} \\ \dot{I}_{m\mathrm{II}} \\ \vdots \\ \dot{I}_{mN} \end{bmatrix} \text{ — matrix-column of the loop currents (MLC);}$$

$$\dot{E}=\begin{bmatrix} \dot{E}_{m\mathrm{I}} \\ \dot{E}_{m\mathrm{II}} \\ \vdots \\ \dot{E}_{mN} \end{bmatrix} \text{ — matrix-column of the loop EMF (MLE).}$$

For solving the system (7.5) we can use

$$\dot{I}_{m\mathrm{I}}=\frac{\Delta_1}{\Delta},\ \dot{I}_{m\mathrm{II}}=\frac{\Delta_2}{\Delta},\ldots,\dot{I}_{mN}=\frac{\Delta_N}{\Delta}, \tag{7.6}$$

where Δ — determinant of the LIM; $\Delta_1,\Delta_2,\ldots,\Delta_N$ — determinants obtained from Δ by replacing the 1-st, 2-nd, ..., N-th column with the matrix-column MLE.

Having determined the loop currents by (7.6), we can calculate currents in the branches from (7.3) and (7.4).

Thus, it is possible to define the following calculation procedure by the loop current method:

1) select the conventionally positive directions of the branch currents;

2) convert all the current sources into equivalent voltage sources;

3) select the independent loops of the network;

4) select the path-tracing directions of the loops: either clockwise or counterclockwise for all;

5) set up a system of loop equations in matrix form for the independent loops;

6) solve the system of equations with respect to the loop currents;

7) express the branch currents in terms of loop currents;

8) using Kirchhoff's current law, find the currents in those branches of the original circuit that were transformed during the transformation of the current sources into the voltage sources.

The advantage of the loop current method: as the equations are set up only according to Kirchhoff's voltage law, the necessary number of equations is less by $q-1$ than in the Kirchhoff equation method.

7.5. The Node Voltage Method

The node voltage method is also considered to be a method of determining variables. The determining variables in terms of which the branch currents in the network are then calculated are node voltages measured for the independent nodes with respect to the basic one.

The essence of the node voltage method consists in setting up and solving a system of Kirchhoff's current law equations. Such equations are set up for independent nodes.

Consider the electric circuit presented in Fig. 4.3, which has already been analyzed. Convert the voltage sources with the EMF $\dot{E}_{m4},\dot{E}_{m5}$ into current sources:

$$\dot{I}_{m4}=\frac{\dot{E}_{m4}}{Z_4}=Y_4\dot{E}_{m4};\quad \dot{I}_{m4}=\frac{\dot{E}_{m5}}{Z_5}=Y_5\dot{E}_{m5}.$$

Also convert the branch impedances into admittances:

$$Y_1=\frac{1}{Z_1};\ Y_2=\frac{1}{Z_2};\ Y_3=\frac{1}{Z_3};\ Y_4=\frac{1}{Z_4};\ Y_5=\frac{1}{Z_5};\ Y_6=\frac{1}{Z_6}.$$

Denote the nodes by I–III, 0. We will get the converted diagram (Fig. 7.6). Construct a graph of this diagram (Fig. 7.7). In so doing, we will take the voltages between the nodes as the branches of the graph. The voltages $\dot{U}_{m1},\dot{U}_{m2},\dot{U}_{m3}$ between nodes I, II, III and node 0 (Fig. 7.6) are denoted by $\dot{U}_{m\mathrm{I}},\dot{U}_{m\mathrm{II}},\dot{U}_{m\mathrm{III}}$ respectively (Fig. 7.7).

Select the tree of the graph through the branches $\dot U_{mI}, \dot U_{mII}, \dot U_{mIII}$ (solid lines). Point out the chords of the graph $\dot U_{m4}, \dot U_{m5}, \dot U_{m6}$ (dotted lines).

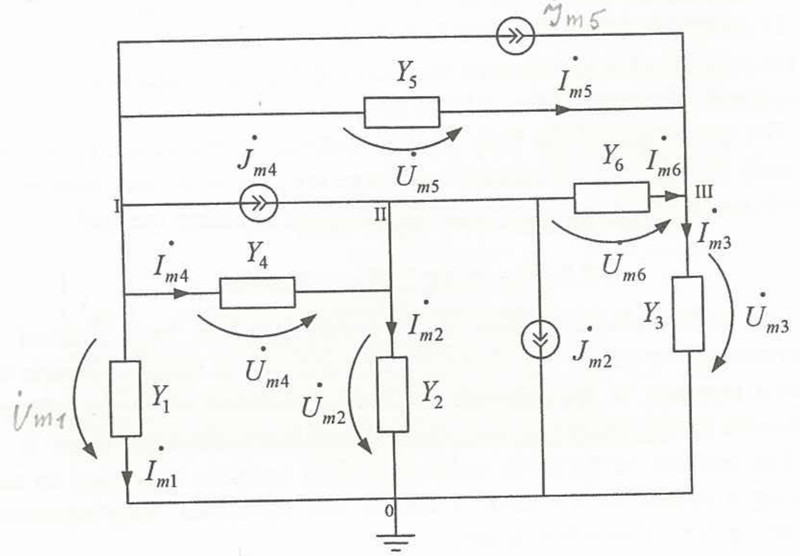


Fig. 7.6

We can see from the graph of Fig. 7.7 that the branch voltages $\dot U_{m1} - \dot U_{m6}$ can be expressed in terms of the voltages $\dot U_{mI}, \dot U_{mII}, \dot U_{mIII}$:

$$\begin{cases} \dot U_{m1} = \dot U_{mI};\ \dot U_{m2} = \dot U_{mII};\ \dot U_{m3} = \dot U_{mIII}; \\ \dot U_{m4} = \dot U_{mI} - \dot U_{mII};\ \dot U_{m5} = \dot U_{mI} - \dot U_{mIII};\ \dot U_{m6} = \dot U_{mII} - \dot U_{mIII}. \end{cases} \tag{7.7}$$

Nodes I – III of the networks art called independent nodes, node 0 – the basic node. The voltages $\dot U_{mI}, \dot U_{mII}, \dot U_{mIII}$ between the independent nodes and the basic one are called node voltages. We can see from (7.7) that the branch voltages can be expressed in terms of node voltages.

The number of node voltages, as well as the number of independent nodes, is less than the number of branches of the network. Hence, the order of the system of equations is reduced and the calculation is simplified.

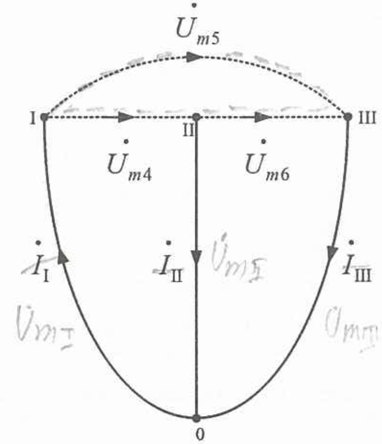


Fig. 7.7

Let us determine the node voltages in the network of Fig. 7.6. Kirchhoff's current law equations for the nodes I – III can be written as

$$\begin{cases} Y_1\dot U_{mI} + Y_4(\dot U_{mI} - \dot U_{mII}) + Y_5(\dot U_{mI} - \dot U_{mIII}) + \dot J_{m4} + \dot J_{m5} = 0; \\ -Y_4(\dot U_{mI} - \dot U_{mII}) + Y_2\dot U_{mII} + Y_6(\dot U_{mII} - \dot U_{mIII}) + \dot J_{m2} - \dot J_{m4} = 0; \\ -Y_6(\dot U_{mII} - \dot U_{mIII}) + Y_3\dot U_{mIII} - Y_5(\dot U_{mI} - \dot U_{mIII}) - \dot J_{m5} = 0 \end{cases}$$

or

$$\begin{cases} (Y_1 + Y_4 + Y_5)\dot U_{mI} - Y_4\dot U_{mII} - Y_5\dot U_{mIII} = -\dot J_{m4} + \dot J_{m5}; \\ -Y_4\dot U_{mI} + (Y_2 + Y_4 + Y_6)\dot U_{mII} - Y_6\dot U_{mIII} = -\dot J_{m2} + \dot J_{m4}; \\ -Y_5\dot U_{mI} - Y_6\dot U_{mII} + (Y_3 + Y_5 + Y_6)\dot U_{mIII} = \dot J_{m5}. \end{cases} \tag{7.8}$$

The system (7.8) can be rewritten in the form:

$$\begin{cases} Y_{11}\dot U_{mI} + Y_{12}\dot U_{mII} + Y_{13}\dot U_{mIII} = \dot J_{mI}; \\ Y_{21}\dot U_{mI} + Y_{22}\dot U_{mII} + Y_{23}\dot U_{mIII} = \dot J_{mII}; \\ Y_{31}\dot U_{mI} + Y_{32}\dot U_{mII} + Y_{33}\dot U_{mIII} = \dot J_{mIII}. \end{cases}$$

Here the values

$$Y_{11} = Y_1 + Y_4 + Y_5,\ Y_{22} = Y_2 + Y_4 + Y_6,\ Y_{33} = Y_3 + Y_5 + Y_6$$

are called self-admittances of the I-st, II-nd and III-rd nodes respectively. Thus, the self-admittance of a node represents the sum of the admittances of the branches converging to this node. The values

$$Y_{12} = Y_{21} = -Y_4,\ Y_{13} = Y_{31} = -Y_5,\ Y_{23} = Y_{32} = -Y_6$$

are called mutual admittances between the I-st and II-nd, the I-st and III-th, and the II-nd and III-th nodes respectively. Thus, the mutual admittance between two nodes is the sum of the admittances of the branches connecting these nodes and taken with the inverse sign.

The currents

$$\dot{J}_{mII} = -\dot{J}_{m4} - \dot{J}_{m5};\ \dot{J}_{mII} = -\dot{J}_{m2} + \dot{J}_{m4};\ \dot{J}_{mIII} = \dot{J}_{m5}$$

are called nodal currents of the I-st, II-nd and III-rd nodes respectively. They represent the sum of the currents of the current sources converging to a given node and taken with a plus sign if the currents are directed to the node, and with a minus sign if the currents are directed from the node.

In the general case of N independent nodes, we can write the following equation system:

$$\begin{cases} Y_{11}\dot{U}_{mI} + Y_{12}\dot{U}_{mII} + \cdots + Y_{1N}\dot{U}_{mN} = \dot{J}_{mI}; \\ Y_{21}\dot{U}_{mI} + Y_{22}\dot{U}_{mII} + \cdots + Y_{2N}\dot{U}_{mN} = \dot{J}_{mII}; \\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots \\ Y_{N1}\dot{U}_{mI} + Y_{N2}\dot{U}_{mII} + \cdots + Y_{NN}\dot{U}_{mN} = \dot{J}_{mN}. \end{cases}$$

Or, in matrix form

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \cdot \begin{bmatrix} \dot{U}_{mI} \\ \dot{U}_{mII} \\ \vdots \\ \dot{U}_{mN} \end{bmatrix} = \begin{bmatrix} \dot{J}_{mI} \\ \dot{J}_{mII} \\ \vdots \\ \dot{J}_{mN} \end{bmatrix}, \qquad (7.9)$$

that is

$$Y\dot{U} = \dot{I},$$

where

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix}$$ — node admittance matrix (NAM);

$$\dot{U} = \begin{bmatrix} \dot{U}_{mI} \\ \dot{U}_{mII} \\ \vdots \\ \dot{U}_{mN} \end{bmatrix}$$ — matrix-column of nodal voltages (MNV);

$$\dot{J} = \begin{bmatrix} \dot{J}_{mI} \\ \dot{J}_{mII} \\ \vdots \\ \dot{J}_{mN} \end{bmatrix}$$ — matrix-column of nodal currents (MNC).

The system (7.9) can be solved by means of determinants

$$\dot{U}_{mI} = \frac{\Delta_1}{\Delta},\ \dot{U}_{mII} = \frac{\Delta_2}{\Delta},\ \ldots,\ \dot{U}_{mN} = \frac{\Delta_N}{\Delta},$$

where Δ — determinant of NAM; $\Delta_1, \Delta_2, \ldots, \Delta_N$ — determinants obtained by replacing the 1-st, 2-nd, ... , N-th columns by the matrix-column MNC.

Having determined the node voltages from (7.7), we can calculate the branch voltages and then the branch currents.

Thus, the following procedure can be suggested for calculation using the node voltage method:

1) select the conventionally positive direction of currents in the branches;

2) convert all voltage sources into equivalent current sources, and resistances — into conductances;

3) select the conventionally positive direction of the branch voltages;

4) select the independent nodes of the circuit, indicate the basic node;

5) set up a system of node equations in matrix form for the independent nodes;

6) solve the system of node equations with respect to the node voltages;

7) express the branch voltages in terms of node voltages;

8) find the branch currents via branch voltages;

9) using Kirchhoff's current law, find the currents in the branches of the original circuit that were transformed during the transformation of the voltage sources into the current sources.

The loop current method and the node voltage method are dual.

Example 1

Set up a system of equations using the Kirchhoff equation method for the electric circuit shown in Fig. 7.8.

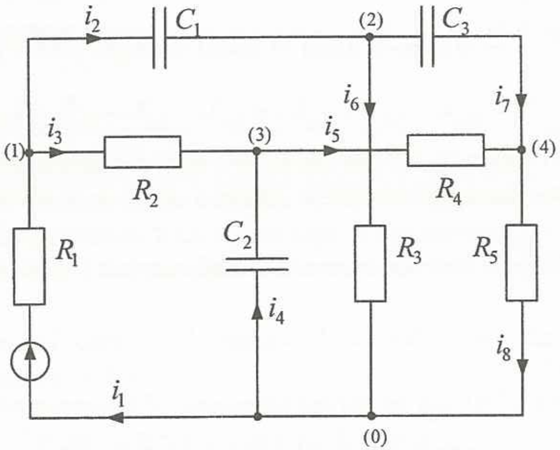


Fig. 7.8

Solution

The circuit has five nodes $(q=5)$, node 0 being the basic, and the other four – independent. The four equations are set up in accordance with Kirchhoff's current law:

for node 1: $I_1 - I_2 - I_3 = 0$;

for node 2: $I_2 - I_6 - I_7 = 0$;

for node 3: $I_3 - I_4 - I_5 = 0$;

for node 4: $I_5 + I_7 - I_8 = 0$.

The circuit consists of eight branches $(p=8)$. Then the number of independent loops is $p-q+1=4$. The other four equations are set up

according to Kirchhoff's voltage law, taking account of the chosen loop tracing directions.

For the loop $E - R_1 - R_2 - C_2 : -E + I_1R_1 + I_3R_2 - I_4\dfrac{1}{j\omega C_2} = 0.$

For the loop $C_2 - R_4 - R_5 : I_4\dfrac{1}{j\omega C_2} + I_5R_4 + I_8R_5 = 0.$

For the loop $E - R_1 - C_1 - R_3 :\ -E + I_1R_1 - I_2\dfrac{1}{j\omega C_2} + I_6R_6 = 0.$

For the loop $R_3 - C_3 - R_5 : -I_6R_3 + I_7\dfrac{1}{j\omega C_3} + I_8R_5 = 0.$

These eight equations constitute the full (consistent) system of equations for the circuit.